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Total $\gamma^\star\gamma^\star$ cross section and the QCD dipole picture

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Abstract. In the framework of the dipole picture of the BFKL pomeron we discuss two possibilities of calculating the total $\gamma^\star\gamma^\star$ cross section of the virtual photons. It is shown that the dipole model reproduces the results obtained earlier from k_T-factorization up to the selection of the scale determining the length of the QCD cascade. The choice of scale turns out to be important for the numerical outcome of the calculations.

1 Introduction

Testing the BFKL pomeron through collisions of tagged e^+e^- pairs with very large momentum transfers is an attractive possibility which has already been discussed in [1,2]. Clearly, the crucial ingredient in the expression for the e^+e^- total cross section at fixed Q_A^2, Q_B^2 momentum transfers of the tagged leptons is the total cross section of two virtual photons of "masses" Q_A^2 and Q_B^2. Calculation of this cross section, $\sigma_{\gamma\gamma}$, is basic for the content of [1,2].

Here, we present a method of calculating $\sigma_{\gamma\gamma}$, alternative to the one presented in [1] and [2], which is an implementation of the dipole picture of the BFKL pomeron proposed in [3–6]. This implementation goes somewhat beyond the applications of the dipole picture given in [7–12] and is based on a discussion of the scale relevant in collisions of highly asymmetric q-$\bar{\text{q}}$ configurations of light quarks within the framework of Mueller's QCD dipole picture [13].

We start with the forward onium-onium amplitude for a single pomeron exchange, $F^{(1)}$, which we calculated *ab initio*. We believe that this new expression is better than the one used in [9–12]. Its detailed derivation is given in Appendix A and the result is

$$F^{(1)} = \pi\alpha^2 r_A r_B \int \frac{d\gamma}{2\pi i} e^{\Delta(\gamma)Y} \left(\frac{r_A}{r_B}\right)^{\gamma-1} h(\gamma) \ . \qquad (1)$$

Here α is the strong coupling constant, N is the number of colors, r_A and r_B are the transverse sizes of the two colliding onia, $\Delta(\gamma) = \alpha N \chi(\gamma)/\pi$ where

$$\chi(\gamma) = 2\psi(1) - \psi(1 - \tfrac{1}{2}\gamma) - \psi(\tfrac{1}{2}\gamma) \quad \left(\psi \equiv \frac{d\log\Gamma}{d\gamma}\right) \qquad (2)$$

and

$$h(\gamma) = \frac{4}{\gamma^2(2-\gamma)^2} \ . \qquad (3)$$

The quantity

$$Y = \log\left(\frac{s}{s_0}\right) \qquad (4)$$

is the total length of the dipole cascade, *i.e.*, the sum of the cascade lengths of the two colliding onia. s is the total c.m. energy of the collision and s_0 is the relevant scale of the problem. s_0 cannot be calculated within the leading logarithmic approximation and therefore it remains an unknown element in this approach. Its determination must rely on one's physical intuition and on results of a phenomenological analysis of data. In the present paper we explore the consequences of the choice suggested by the dipole picture [13].

While (1) reproduces the saddle point approximation of $F^{(1)}$ derived in [3–5] (and employed in [9–12]), it also contains contributions which are neglected in the contour integral representations of $F^{(1)}$ used in [9–12]. In other words, in [9–12], those components of the integrand which become unity at the saddle point, are kept equal one throughout the whole contour integration. This approximation has been corrected in our present expression for $F^{(1)}$.

The way to employ the dipole picture to calculate the total $\gamma^\star\gamma^\star$ cross section is, in principle, straightforward. From $F^{(1)}$ and the well known (compare, *e.g.*, [6,10,12,14]) wave functions of the two photons, A and B, of the virtual masses $Q_{A,B}$, longitudinally (L) or transversely (T) polarized, $\Psi^{L,T}(r_{A,B}, z_{A,B}; Q_{A,B})$, we obtain the for-

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ward $\gamma^\star\gamma^\star$ amplitude

$$F_{\gamma\gamma} = \int |\Psi^{L,T}(r_A, z_A; Q_A)|^2 \times |\Psi^{L,T}(r_B, z_B; Q_B)|^2 \, F^{(1)} d^2 r_A \, dz_A \, d^2 r_B \, dz_B \,, \quad (5)$$

and the total cross section which, with our conventions, reads

$$\sigma_{\gamma\gamma} = 2 \,\mathrm{Re}\, F_{\gamma\gamma} \,. \quad (6)$$

Then, in order to evaluate the integral in (5), we have to decide what to take for Y, the length of the cascade. According to (4) this amounts to a selection of the scale s_0. Two choices were discussed in the literature. In [1] s_0 was taken as

$$s_0 = c\, Q_A Q_B \,, \quad (7)$$

with $c = 100$. This apparently natural choice has an attractive feature to be a simple analytic function of Q_A and Q_B. Another possibility [7] was to take

$$s_0 = c\, Q_>^2 \,, \quad (8)$$

where $Q_>$ is the larger of Q_A, Q_B. This gives

$$Y = \log\left(\frac{1}{c x_{Bj}}\right), \quad (9)$$

a formula which provided an excellent fit to the proton structure function at small x_{Bj} [7,8][1].

Following [13] we observe, however, that from the point of view of the dipole picture neither (7) nor (8) is really satisfactory. The point is that (1) refers to collisions of two *dipoles* and thus the relevant scale s_0 must be expressed in term of the parameters characterizing these dipoles (*i.e.*, longitudinal momenta z_A, z_B and transverse sizes r_A, r_B) rather than Q_A and Q_B. This is clear if one observes that Q_A and Q_B are not even defined in (1). The possible choices of s_0 consistent with the dipole picture were discussed in [13], where also a definite formula for Y was suggested. In the present paper we explore consequences of this choice for $\sigma_{\gamma\gamma}$ and compare it with the results following from (7) and (8). We hope that our results shall be useful in testing the validity of the dipole picture approach in the small x_{Bj} physics.

In the next section we present our formulae for $\sigma_{\gamma\gamma}$ following from Y worked out in [13]. In Sect. 3 we give our numerical results and their discussion. Sect. 4 contains the conclusions. Appendix A presents a detailed derivation of the formula (1), in Appendix B we calculate $\sigma_{\gamma\gamma}$ for three factorizable forms of Y defined by: (13), (7), and (8). The case (7) gives $\sigma_{\gamma\gamma}$ of [1] whereas the last one, (8), results in $\sigma_{\gamma\gamma}$ which follows from the version of the dipole picture implemented in [7].

[1] Note that (7) and (8) give rather different results for Q^2 dependence of $\gamma - \gamma$ cross section which should not be too difficult to test once the relevant data are available

2 The total $\gamma^\star$-$\gamma^\star$ cross section

In this section we derive the formulae for the total cross section of two virtual gammas using the formula (1) for *onium-onium* forward amplitude derived in Appendix A, with Y taken from [13]

$$Y = y_A + y_B = \log\left(\frac{s z_A^< z_B^< r_A^2 r_B^2}{c\tau_{int}^2}\right), \quad (10)$$

where c is the arbitrary constant of the leading log approximation, $s = 4E_A E_B$ is the square of the total c.m. energy of the colliding virtual photons,

$$z^< = \begin{cases} z & \text{if } z \le \frac{1}{2} \\ 1-z & \text{if } z \ge \frac{1}{2}, \end{cases} \quad (11)$$

and

$$\tau_{\mathrm{int}} = \mathrm{const}\, r_> \quad (12)$$

where $r_>$ is the larger of r_A and r_B. τ_{int} is interpreted (see [13]) as the time needed for the exchanged gluons to travel the necessary distance in the transverse space.

We will confront the results obtained with (12) with the ones one gets replacing (12) by a symmetric expression

$$\tau_{\mathrm{int}}^2 \to \tau^2 = r_A r_B \quad (13)$$

which leads to formulae close to the ones advocated in [1, 2].

To get $\gamma^\star - \gamma^\star$ amplitude we employ now the wave functions of the virtual photons, $\Psi(r_A, z_A; Q_A)$, $\Psi(r_B, z_B; Q_B)$, and calculate $F_{\gamma\gamma}$ of (5), where for the transverse (T) and longitudinal (L) photons we have (compare [6,10,12,14])

$$|\Psi^{T,L}(r,z;Q)|^2 = \Phi^{T,L}(r,z;Q) = \frac{N\alpha_{em} e_f^2}{\pi^2} W^{T,L}(r,z;Q) \,, \quad (14)$$

$$W^T(r,z;Q) = \tfrac{1}{2}[z^2 + (1-z)^2]\hat{Q}^2 K_1^2(\hat{Q}r) \,, \quad (15)$$

$$W^L(r,z;Q) = 2z(1-z)\hat{Q}^2 K_0^2(\hat{Q}r) \,, \quad (16)$$

where $\hat{Q} = \sqrt{z(1-z)}Q$, $\alpha_{em} = 1/137$ and $e_f^2 = 2/3$ (the sum of the squares of the charges of three quarks).

Inserting (10) into (1) and employing (5) we obtain for the total $\gamma^\star\gamma^\star$ cross section

$$\sigma_{\gamma\gamma} = 4(2\pi)^3 \alpha^2 \int_0^\infty dr_A r_A^2 \int_0^{\frac{1}{2}} dz_A \Phi^{T,L}(z_A, r_A; Q_A) \times \int_0^\infty dr_B r_B^2 \int_0^{\frac{1}{2}} dz_B \Phi^{T,L}(z_B, r_B; Q_B) \times \int \frac{d\gamma}{2\pi i} \mathrm{e}^{\Delta(\gamma)Y} \left(\frac{r_A}{r_B}\right)^{1-\gamma} h(\gamma) \,, \quad (17)$$

where $Y = \log\xi$ and $\xi = (s z_A^< z_B^< r_A^2 r_B^2/(c\tau_{\mathrm{int}}^2))$. Note that since Φ^T and Φ^L are invariant against the replacements: $z_{A,B} \to (1 - z_{A,B})$ and $(1 - z_{A,B}) \to z_{A,B}$, we can drop the $^<$ superscripts and integrate over z's as follows $\int_0^1 dz \to 2\int_0^{1/2} dz$.

The expression (17) is comparatively easy to evaluate when, as in (13), $\tau_{\rm int}^2 \to \tau^2 = r_A r_B$ (the case close to the one of [1,2]) because the integrals factorize into integrals over the A and B variables, the integrations over r_A and r_B can be done analytically and we obtain

$$\sigma_{\gamma\gamma} = \frac{8}{\pi}(\alpha N \alpha_{em} e_f^2)^2 \frac{1}{Q_A Q_B} \times \int \frac{d\gamma}{2\pi i}\left(\frac{s}{cQ_AQ_B}\right)^{\Delta(\gamma)}\left(\frac{Q_B}{Q_A}\right)^{1-\gamma} h(\gamma) H^{L,T}(\gamma)\,, \tag{18}$$

where $H^{L,T} = 4^{\Delta(\gamma)} Z^{L,T}(\gamma) S^{L,T}(\gamma) Z^{L,T}(2-\gamma) S^{L,T}(2-\gamma)$ is the product of the functions of γ defined in the Appendix B.

In the case of (12), however, there is no factorization and one has to face a 5-dimensional integration with one of the integrals being a contour integral in the complex plane γ (along a straight line parallel to the imaginary axis). This forces us to use a numerical method for the evaluation of (17). It turns out (see the discussion of our results below) that in the very high energy limit (s/c very large) one can safely use the saddle point approximation for the contour integral

$$\int \frac{d\gamma}{2\pi i} \mathrm{e}^{\Delta(\gamma) Y}\left(\frac{r_A}{r_B}\right)^{1-\gamma} h(\gamma) = \tfrac{1}{2}\sqrt{\frac{2a_\xi}{\pi}}\, h(\gamma_0)\, \xi^{\Delta_p} \mathrm{e}^{-\frac{1}{2}a_\xi \log^2(r_A/r_B)}\,, \tag{19}$$

with $Y = \log \xi$, $\xi = s/s_0$, with s_0 as the case might be (see above), $a_\xi = [7\alpha N \zeta(3)\log(\xi)/\pi]^{-1}$ and $\Delta_p = \frac{\alpha N}{\pi}\chi(1)$. $\gamma_0 = 1 - a_\xi \log(r_A/r_B)$ is the saddle point which, in the limit $s/c \to \infty$, equals 1. Then the numerical integration reduces to 4 dimensions. Note that (19) exhibits the source of the substantial difference in dependences on Q_B/Q_A following from (7) and (8), see Fig. 2. This difference sits in the ξ^{Δ_p} factor:

$$\left(\frac{s}{cQ_AQ_B}\right)^{\Delta_p} \quad \text{against} \quad \left(\frac{s}{cQ_>^2}\right)^{\Delta_p}. \tag{20}$$

3 Numerical results and discussion

We considered 4 cases of $Y = \log(s/s_0)$ and calculated the corresponding cross sections:

(a) The case of (7), $s_0 = c_{(a)} Q_A Q_B$, employed in [1,2],
(b) the case of (8), $s_0 = c_{(b)} Q_>^2$, employed in [7],
(c) the case of (12), $s_0 = (c_{(c)} r_>^2/(z_A^< z_B^< r_A^2 r_B^2))$, discussed in [13],
(d) the case of (13), $s_0 = (c_{(d)}/(z_A^< z_B^< r_A r_B))$, discussed also in [13].

Comments: As shown in Appendix B, in the Case (a) we obtain the same formula for $\sigma_{\gamma\gamma}$ as in [1]. Also let us note that, when $Q_A = Q_B$ and the arbitrary constants are set to the same value $c_{(a)} = c_{(b)} = c$, the σ's for Case (a) and Case (b) are identical.

The Cases (a)–(d) were calculated in the saddle point approximation given by the formula (19) and subsequent 4-dimensional integration. In Cases (a), (b) and (d), we checked the accuracy of this procedure calculating $\sigma_{\gamma\gamma}$ analytically up to the final contour integration over γ which was done with the help of MATHEMATICA. It turned out that the results of these two procedures agree to within 15 percent.

In order to exhibit the asymmetries when $Q_A \neq Q_B$ we introduced the asymmetry parameter, ζ, defined as

$$\zeta = \frac{Q_B}{Q_A}. \tag{21}$$

From the asymptotic forms (56) and (64) we see that the asymmetry in Q_A, Q_B in Cases (a) and (d) is given approximately by the factor

$$\mathrm{e}^{-\frac{1}{2}a_\xi \log^2(\zeta)}. \tag{22}$$

In the (b) and (c) cases, especially in Case (b), this estimate is not good enough.

Clearly, the choices of the values of the arbitrary constant c involved in all Y's discussed in this paper are very important in determining the size of the cross section. They can either be fitted to experimental results (compare [7]) or set following some prejudices of the authors (compare, *e.g.*, [1]): in [1]

$$c = c_{(a)} = 100, \quad \xi = \xi_{(a)} = \frac{s}{c_{(a)} Q_A Q_B} \tag{23}$$

and in [7][2]

$$c = c_{(b)} = 0.57, \quad \xi = \xi_{(b)} = \frac{s}{c_{(b)} Q_>^2}\,. \tag{24}$$

The constants c for Cases (c) and (d) were set to fit the σ's of Cases (b) and (a), respectively, for $Q_A = Q_B = 4\,\mathrm{GeV}$ and $\sqrt{s} = 200\,\mathrm{GeV}$. They come out to be: $c_{(c)} = 0.0055$, and $c_{(d)} = 2.5$.

To estimate the role of c's it is enough to use the asymptotic formula (64) for $Q_A = Q_B$. We get

$$\frac{\sigma_{\gamma\gamma}(a)}{\sigma_{\gamma\gamma}(b)} = \left(\frac{\xi_{(a)}}{\xi_{(b)}}\right)^{\Delta_p} \sqrt{\frac{a_{\xi_{(a)}}}{a_{\xi_{(b)}}}} \tag{25}$$

where a_ξ is given below (19). Since, in the limit $s \to \infty$, $\mathcal{O}(\log \xi_{(a)}) = \mathcal{O}(\log \xi_{(b)})$, we have approximately

$$\frac{\sigma_{\gamma\gamma}(a)}{\sigma_{\gamma\gamma}(b)} = \left(\frac{c_{(b)}}{c_{(a)}}\right)^{\Delta_p}. \tag{26}$$

In Fig. 1 we present the $\sigma_{\gamma\gamma}$'s for Cases (a)–(d), for $Q_A = Q_B$, setting the strong coupling constant $\alpha = 0.11$, hence $\Delta_p = 0.3$. The values of the constants c were taken as in [1] ($c_{(a)} = 100$) and [7] ($c_{(b)} = 0.57$). The resulting

[2] The authors are grateful to R. Peschanski and Ch. Royon for providing them with $c_{(b)}$ of (24) which gives the fit of [7]

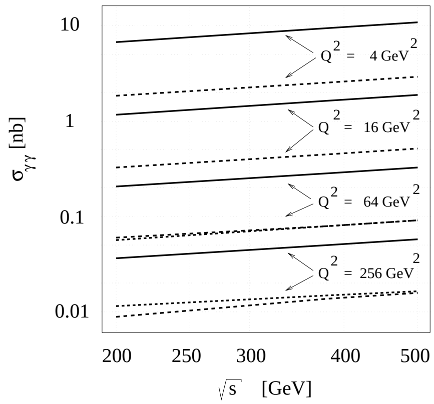

Fig. 1. Total cross section in the dominant TT channel for $Q_A = Q_B = Q$. The *solid lines* represent the results for Cases (b) and (c), whereas the *dashed lines* show Cases (a) and (d). Using the values of the scale parameter c given in the text, one finds that Cases (b) and (c) coincide in the wide range of Q^2. The results for Cases (a) and (d) slightly differ for very large Q^2

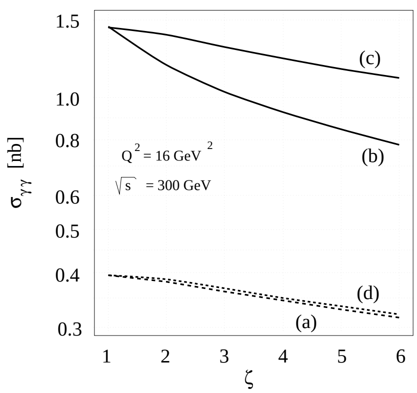

Fig. 2. Total cross section in the dominant TT channel for $1 \leq \zeta \leq 6$

cross sections differ appreciably, consistently with (26). The dipole model results, (c) and (d), were fitted to the predictions of (b) and (a), respectively, at the point $Q^2 = 16\,\mathrm{GeV}^2$ and $\sqrt{s} = 200\,\mathrm{GeV}$. One sees that they follow closely the results of (a) and (b) for all considered values of Q^2 and $\sqrt{s}$.

In Fig. 2 the dependence on the asymmetry parameter $\zeta = Q_B/Q_A$ is plotted. Other parameters are chosen as in Fig. 1. One sees that the ζ-dependence is almost identical for two versions of the dipole model ((c) and (d)) and the symmetric proposal of [1,2]. The case (b) differs significantly, however, from the others, giving a much stronger dependence on ζ.

4 Conclusions

The predictions of the dipole model for the photon - photon cross section depend strongly on the scale determining the length of the dipole cascade in the incident photons. The scale suggested previously [1,2] gives substantially smaller cross section than the one suggested by a fit of the dipole model results to the proton structure function [7]. On the other hand, the dependence of the cross section on the ratio Q_B/Q_A for the two colliding photons turned out to be the same for the two extreme cases of the dipole model ((c), (d)) suggesting that it hardly depends on the details of the model.

We conclude that future measurements of $\gamma^*\gamma^*$ cross section may be useful in determining the length of the dipole (gluon) cascade but, probably, not very helpful in understanding the details of the dipole-dipole interaction. This makes rather urgent the need of determining the relevant scales from the higher-order perturbative calculations.

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Appendix A Forward onium-onium amplitude including the light-cone momentum fractions dependencies

In this Appendix we give a direct derivation of the formula for $F^{(1)} = \int d^2b F^{(1)}(b)$ in the form of a contour integral. This derivation accommodates the option that the dipole-dipole cross section takes the asymptotic form only above some energy E_0 of the slowest gluon.

We follow the notation of the main text: the subscripts A and B refer to one of the virtual photons. For example, x_A is the transverse size of a dipole originating from the virtual photon A. Also $x_<$ denotes the smaller of the two sizes x_A and x_B. As before, $r_{A,B}$ and $z_{A,B}$ are the arguments of the photon wave functions. Remember, however, that $z^<_{A,B}$ are defined by (11). Consulting of [13] may also be helpful.

We start from the formula (3) of Mueller's paper [5] which we write in the form

$$F^{(1)} = \pi\alpha^2 \int \frac{dx_A}{x_A}\frac{dx_B}{x_B} d^2s\,\sigma(x_A,x_B) \times n(r_A,x_A,y_A,s)\,n(r_B,x_B,y_B,b-s)\,, \quad (27)$$

where in the limit of high energy

$$\sigma(x_A,x_B) = 4\int \frac{dl}{l^3}[1-J_0(x_A l)][1-J_0(x_B l)] = x_<^2[1+\log(x_>/x_<)]\,, \quad (28)$$

and

$$n(r,x,y,s) = \frac{1}{\pi^2}\int \frac{d\gamma}{2\pi i} \mathrm{e}^{\Delta(\gamma)y}(1-\gamma)^2\left(\frac{r}{x}\right)^{\gamma} I(r,x,s,\gamma)\,, \quad (29)$$

with

$$I(r,x,s,\gamma) = \int d^2w\,\left(|s+\tfrac{1}{2}x-w||s-\tfrac{1}{2}x-w|\right)^{\gamma-2} \times \left(|\tfrac{1}{2}r-w||-\tfrac{1}{2}r-w|\right)^{-\gamma}\,. \quad (30)$$

Following Mueller we also find (see [13])

$$y = \log\left(\frac{z_<}{z_0}\right), \tag{31}$$

where $z_<$ is defined in (11) and z_0 is the minimal fraction of the onium energy which can be carried by a dipole in the wave function (actually the minimal energy carried by one of the gluons forming a dipole). Mueller does not explain why the integration must stop at some z_0 (except for the obvious fact that for $z_0 = 0$ the formula diverges). It is clear, however, that without understanding this problem there is no chance to estimate reasonably z_0. Following the arguments of [13] (*e.g.*, (18) and (21)) we take

$$z_0 = \frac{p_0^+}{p^+} = \frac{\tau_{int}}{r^2 p^+}, \tag{32}$$

where $p_0^+ = 2E_0$, E_0 being the energy of the slowest gluon.

When all this is substituted into (1) we obtain

$$\begin{aligned} F^{(1)} = \frac{\alpha^2}{\pi^3}\int \frac{d\gamma_A}{2\pi i}\frac{d\gamma_B}{2\pi i}\left(\frac{z_<^A p_A^+ r_A^2}{\tau_{int}}\right)^{\Delta(\gamma_A)}(1-\gamma_A)^2 \quad (33)\\ \times\left(\frac{z_<^B p_B^+ r_B^2}{\tau_{int}}\right)^{\Delta(\gamma_B)}(1-\gamma_B)^2 \Omega(r_A, r_B, \gamma_A, \gamma_B), \end{aligned}$$

where

$$\begin{aligned} \Omega(r_A, r_B, \gamma_A, \gamma_B) = \int \frac{dx_A}{x_A}\frac{dx_B}{x_B}\sigma(x_A, x_B)\left(\frac{r_A}{x_A}\right)^{\gamma_A} \quad (34)\\ \times\left(\frac{r_B}{x_B}\right)^{\gamma_B}\tilde{I}(x_A, x_B, r_A, r_B, \gamma_A, \gamma_B), \end{aligned}$$

with

$$\begin{aligned} &\tilde{I}(x_A, x_B, r_A, r_B, \gamma_A, \gamma_B)\\ &= \int d^2b\, d^2s\, d^2w\, d^2u\\ &\times\left(|b-s+\tfrac{1}{2}x_B-u||b-s-\tfrac{1}{2}x_B-u|\right)^{\gamma_B-2}\\ &\times\left(|\tfrac{1}{2}r_B-u||-\tfrac{1}{2}r_B-u|\right)^{-\gamma_B}\\ &\times\left(|s+\tfrac{1}{2}x_A-w||s-\tfrac{1}{2}x_A-w|\right)^{\gamma_A-2}\\ &\times\left(|\tfrac{1}{2}r_A-w||-\tfrac{1}{2}r_A-w|\right)^{-\gamma_A}. \end{aligned} \tag{35}$$

After the change of variables, $s-w \to s$, $b-s-u \to b$, (35) factorizes:

$$\begin{aligned} &\tilde{I}(x_A, x_B, r_A, r_B, \gamma_A, \gamma_B) \quad (36)\\ &= I(x_B, 2-\gamma_B)\, I(x_A, 2-\gamma_A)\, I(r_A, \gamma_A)\, I(r_B, \gamma_B), \end{aligned}$$

where

$$I(x,\lambda) = \int d^2s(|s+\tfrac{1}{2}x||s-\tfrac{1}{2}x|)^{-\lambda}. \tag{37}$$

$I(x,\lambda)$ can be calculated using the technique of Mueller. Here we give only the result. It reads

$$I(x,\lambda) = \pi x^{2(1-\lambda)}H(\lambda), \tag{38}$$

where

$$H(\lambda) = \frac{\Gamma^2(1-\frac{1}{2}\lambda)\Gamma(\lambda-1)}{\Gamma^2(\frac{1}{2}\lambda)\Gamma(2-\lambda)}. \tag{39}$$

Consequently, we can write

$$\begin{aligned} \Omega(r_A, r_B, \gamma_A, \gamma_B) = \pi^4 H(2-\gamma_A)H(2-\gamma_B)H(\gamma_A)H(\gamma_B)\\ \times r_A^{2-\gamma_A} r_B^{2-\gamma_B}\omega(\gamma_A, \gamma_B), \end{aligned} \tag{40}$$

where

$$\omega(\gamma_A, \gamma_B) = \int_0^\infty \frac{dx_A}{x_A}\frac{dx_B}{x_B}x_A^{\gamma_A-2}x_B^{\gamma_B-2}\sigma(x_A, x_B). \tag{41}$$

Integration over x_A and x_B is tedious but we do it explicitly

$$\begin{aligned} &\omega(\gamma_A, \gamma_B)\\ &= \int_0^\infty \frac{dx_A}{x_A}x_A^{\gamma_A-2}\left[\int_0^{x_A}\frac{dx_B}{x_B}(x_B)^{\gamma_B}(1+\log\frac{x_A}{x_B})\right.\\ &\quad\left. + x_A^2\int_{x_A}^\infty \frac{dx_B}{x_B}(x_B)^{\gamma_B-2}(1+\log\frac{x_B}{x_A})\right]. \end{aligned} \tag{42}$$

The integrals over dx_B are well defined and are:

$$\int_0^{x_A}\frac{dx_B}{x_B}(x_B)^{\gamma_B}\left(1+\log\frac{x_A}{x_B}\right) = x_A^{\gamma_B}\frac{1+\gamma_B}{(\gamma_B)^2}, \tag{43}$$

$$x_A^2\int_{x_A}^\infty \frac{dx_B}{x_B}(x_B)^{\gamma_B-2}\left(1+\log\frac{x_B}{x_A}\right) = x_A^{\gamma_B}\frac{3-\gamma_B}{(2-\gamma_B)^2}. \tag{44}$$

When this is substituted into (42) and integrated over dx_A we obtain

$$\begin{aligned} \omega(\gamma_A, \gamma_B) &= h(\gamma_B)\int_0^\infty \frac{dx_A}{x_A}x_A^{\gamma_A+\gamma_B-2}\\ &= \frac{h(\gamma_B)\epsilon^{\gamma_A+\gamma_B-2}}{2-\gamma_A-\gamma_B}, \end{aligned} \tag{45}$$

where we have regularized the integral by taking the lower limit to be ϵ. $h(\gamma_B)$ is given by

$$h(\gamma_B) = \frac{4}{(2-\gamma_B)^2(\gamma_B)^2}. \tag{46}$$

Inserting (39) and (45) into (40) we obtain

$$\begin{aligned} &(1-\gamma_A)^2(1-\gamma_B)^2\Omega(r_A, r_B, \gamma_A, \gamma_B)\\ &= \pi^4 r_A^{2-\gamma_A} r_B^{2-\gamma_B}\frac{h(\gamma_B)\epsilon^{\gamma_A\gamma_B-2}}{2-\gamma_A-\gamma_B}. \end{aligned} \tag{47}$$

Now, it turns out that the result for $F^{(1)}$ depends on the order of integration over γ's. The integral over γ_B in (33) can be calculated just by taking the residue at the pole $(2-\gamma_A-\gamma_B)^{-1}$. We obtain

$$F^{(1)} = \pi\alpha^2 r_A r_B \int \frac{d\gamma}{2\pi i}\mathrm{e}^{\Delta(\gamma)Y}\left(\frac{r_A}{r_B}\right)^{1-\gamma}h(\gamma), \tag{48}$$

where

$$Y = \log(p_A^+ p_B^+ z_<^A z_<^B r_A^2 r_B^2/\tau_{int}^2) = \log(s z_<^A z_<^B r_A^2 r_B^2/\tau_{int}^2)\,, \tag{49}$$

and $s = p_A^+ p_B^+$ is the total c.m. energy squared. The integration over γ_A can be similarly executed and one obtains for $F^{(1)}$ again the result (48) but with $r_A, r_B \to r_B, r_A$ exchanged. However, (48) is in fact invariant under this exchange (see, *e.g.*, the asymptotic form of $F^{(1)}$ (19) which exhibits such an invariance).

Appendix B

As it was pointed out in [13], the dipole picture of BFKL [3–5] gives some freedom of choice of the specific realizations of Y in (1). In this Appendix we give derivations of $\sigma_{\gamma\gamma}$ for two cases of the explicitly symmetric (with respect to the virtual photons A and B) expressions for Y.

We start with the case of $\tau_{\rm int}^2$ given by (13). Now $F^{(1)}$ becomes

$$F^{(1)} = \pi\alpha^2 r_A r_B \int \frac{d\gamma}{2\pi i}(s r_A r_B z_A^< z_B^</c)^{\Delta(\gamma)} \left(\frac{r_A}{r_B}\right)^{\gamma-1} h(\gamma)\,, \tag{50}$$

From (5), (14), (15) and (16) it follows that the integrations over r_A, r_B, z_A, and z_B factorize. The integrals over r_A and r_B can be done with the help of the formula 6.576/4 of Gradstein and Ryzhik [15], and one obtains after a few straightforward operations

$$\begin{aligned}\sigma_{\gamma\gamma} &= \frac{8}{\pi}(\alpha N \alpha_{em} e_f^2)^2 \frac{1}{Q_A Q_B} \\ &\times \int \frac{d\gamma}{2\pi i} h(\gamma) \left(\frac{4s}{cQ_A Q_B}\right)^{\Delta(\gamma)} \left(\frac{Q_B}{Q_A}\right)^{1-\gamma} \\ &\times Z_A^{L,T}(\gamma) S_A^{L,T}(\gamma) Z_B^{L,T}(2-\gamma) S_B^{L,T}(2-\gamma)\,,\end{aligned} \tag{51}$$

where

$$S^T(\gamma) = \frac{4-\gamma_-}{2-\gamma_-} S^L(\gamma)\,, \quad S^L(\gamma) = \frac{\Gamma^4(2-\frac{1}{2}\gamma_-)}{\Gamma(4-\gamma_-)}\,, \tag{52}$$

$$Z^T(\gamma) = \int_0^{\frac{1}{2}} dz[z^2 + (1-z)^2] z^{\frac{1}{2}\gamma_+ - 1}(1-z)^{\frac{1}{2}\gamma_- - 1}\,, \tag{53}$$

$$Z^L(\gamma) = 4\int_0^{\frac{1}{2}} dz z^{\frac{1}{2}\gamma_+}(1-z)^{\frac{1}{2}\gamma_-}\,, \tag{54}$$

and

$$\gamma_\pm = \gamma \pm \Delta(\gamma)\,. \tag{55}$$

It is also of interest to have the asymptotic (*i.e.*, $s/c \to \infty$) expression for $\sigma_{\gamma\gamma}$. This can be done through the saddle point approximation around $\gamma = 1$ (from (2) we have that $\chi'(1) = 0$). We obtain

$$\begin{aligned}\sigma_{\gamma\gamma} &= \frac{16}{\pi}(\alpha N \alpha_{em} e_f^2)^2 \frac{1}{Q_A Q_B}\sqrt{\frac{2a_\xi}{\pi}}\xi^{\Delta_p} \\ &\times e^{-\frac{1}{2}a_\xi \log^2(Q_B/Q_A)}[Z^{L,T}(1) S^{L,T}(1)]^2\,,\end{aligned} \tag{56}$$

where

$$\xi = \frac{4s}{cQ_A Q_B}\,, \quad \Delta_P = \Delta(\gamma = 1)\,,$$

$$a_\xi = [7\alpha N \zeta(3)\log(\xi)/\pi]^{-1}\,, \tag{57}$$

$$S^T(1) = S^L(1)\frac{3+\Delta_P}{1+\Delta_P}\,, \quad S^L(1) = \frac{\Gamma^4(\frac{3}{2}+\frac{1}{2}\Delta_P)}{\Gamma(3+\Delta_P)}\,, \tag{58}$$

$$Z^T(1) = \int_0^{\frac{1}{2}} dz[z^2+(1-z)^2] z^{\frac{1}{2}(\Delta_P - 1)}(1-z)^{-\frac{1}{2}(\Delta_P+1)}\,, \tag{59}$$

$$Z^L(1) = 4\int_0^{\frac{1}{2}} dz z^{\frac{1}{2}(1+\Delta_P)}(1-z)^{\frac{1}{2}(1-\Delta_P)}\,. \tag{60}$$

To have a direct comparison of the dipole picture formulae with those of [1,2] and with [7], we employ the forms of Y which do not depend on r's and z's ((4) and (7), (4) and (8)). Now all integrals over r_A , r_B, z_A, z_B can be done analytically employing the formula 6.576/4 of [15] for the integrals over r's, and the integrals over z's are now simple Euler β functions. We obtain

$$\begin{aligned}\sigma_{\gamma\gamma} &= \frac{32\alpha^2 N^2 \alpha_{em}^2 (e_f^2)^2}{\pi Q_A Q_B} \\ &\times \int \frac{d\gamma}{2\pi i}\xi^{\Delta(\gamma)}(Q_A/Q_B)^{1-\gamma}\frac{W_A^{L,T}(2-\gamma) W_B^{L,T}(\gamma)}{(2-\gamma)^2\gamma^2}\,,\end{aligned} \tag{61}$$

where

$$W^T(\gamma) = \frac{4-\gamma}{2-\gamma}\frac{\Gamma^4(1+\frac{1}{2}\gamma)}{\Gamma(2+\gamma)}\frac{\Gamma(3-\frac{1}{2}\gamma)\Gamma(1-\frac{1}{2}\gamma)}{\Gamma(4-\gamma)}\,, \tag{62}$$

$$W^L(\gamma) = 2\frac{\Gamma^4(1+\frac{1}{2}\gamma)}{\Gamma(2+\gamma)}\frac{\Gamma^2(2-\frac{1}{2}\gamma)}{\Gamma(4-\gamma)}\,. \tag{63}$$

Setting $\xi = s/(cQ_A Q_B)$ we obtain $\sigma_{\gamma\gamma}$ of [1]. The transition between (8) of [1] and (61) goes through the substitution $\gamma = 2\gamma'$ which changes χ of (2) and transforms the r.h.s. of (61) into the r.h.s. of (8) in [1], multiplied by a factor $\frac{8}{9}$. This factor can be traced to an approximation made in [5].

For the sake of completeness we give also the saddle point approximation of (61)

$$\begin{aligned}\sigma_{\gamma\gamma} &= \frac{16\alpha^2 N^2 \alpha_{em}^2 (e_f^2)^2}{\pi Q_A Q_B}\xi^{\Delta_p}\sqrt{\frac{2a_\xi}{\pi}} \\ &\times e^{-\frac{1}{2}a_\xi \log^2(Q_A/Q_B)} W_A(1) W_B(1)\,,\end{aligned} \tag{64}$$

where

$$W^T(1) = \frac{9\pi^3}{256}\,, \quad W^L(1) = \frac{2\pi^3}{256}\,. \tag{65}$$

Note that for the case discussed in [7] we have $\xi = s/(cQ_>^2)$ (compare (8), and the same set of formulae for $\sigma_{\gamma\gamma}$ (61)–(64)).

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